Influence of the angular shape of the volume-scattering function and multiple scattering on remote sensing reflectance

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Scattering phase functions derived from measured (volume-scattering meter, VSM) volume-scattering functions (VSFs) from Crimean coastal waters were found to have systematic differences in angular structure from Fournier–Forand (FF) functions with equivalent backscattering ratios. Hydrolight simulations demonstrated that differences in the angular structure of the VSF could result in variations in modeled subsurface radiance reflectances of up to \( \pm 20\% \). Furthermore, differences between VSM and FF simulated reflectances were found to be nonlinear as a function of scattering and could not be explained with the single-scattering approximation. Additional radiance transfer modeling demonstrated that the contribution of multiple scattering to radiance reflectance increased exponentially from a minimum of 16\% for pure water to a maximum of \( \pm 94\% \) for turbid waters. Monte Carlo simulations demonstrated that multiple forward-scattering events were the dominant contributors to the generation of radiance reflectance signals for turbid waters and that angular structures in the shape of the VSF at forward angles could have a significant influence in determining reflectance signals for turbid waters. © 2006 Optical Society of America

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1. Introduction

Relatively few measurements of the volume-scattering function (VSF), \( \beta(\theta) \), have been made since the pioneering work done before and during the early 1970s.\(^1\)–\(^4\) Measuring the VSF is notoriously challenging owing to the technical difficulties in designing adequate instrumentation. Recently there have been significant attempts to address this information shortfall, notably by Volten \textit{et al.}\(^5\) who looked at VSFs for various algal cultures and sediment samples and Voss \textit{et al.}\(^6\) who resurrected the original Petzold instrumentation and measured the volume-scattering function for samples of the coccolithophorid \textit{Emiliania huxleyi}. In this paper we present VSF data from a novel instrument designed by the Marine Hydrophysical Institute of the Ukrainian National Academy of Sciences. The volume-scattering meter (VSM) has previously been found to provide very high quality data over a full range of scattering angles from 0.7° to 178° backed up by rigorous calibration.\(^7\)–\(^11\) Access to measured VSFs allows a comparison between true scattering phase functions [hereafter referred to as \( \beta(\theta) \)], including detailed angular structures, with analytic approximations. In this paper we shall focus on comparisons between scattering phase functions derived from the VSM data and Fournier–Forand (FF) functions with the same backscattering ratio. The FF approximation was selected for the following reasons: (i) it is based on a physical model of marine particle populations,\(^12\)\(^,\)\(^13\) (ii) it has previously been found to give reasonably realistic values for radiance transfer simulations,\(^14\) and (iii) it is currently used in the most recent versions of the HYDROLIGHT (Sequoia Scientific, Inc., Redmond, Wash.) radiance transfer software package.

In this paper we shall examine the sensitivity of remote sensing reflectance signals to angular structure in the shape of the VSF (and by implication the scattering phase function). Our analysis will be based on simulated radiance reflectance signals from im-
mediately beneath the sea surface, defined as \( R_L(\theta) = L_u(\theta)/E_d \) where \( L_u(\theta) \) is angularly resolved upward radiance (\( \theta \) is the scattering angle), and \( E_d \) is downward irradiance.

Various models describing radiance reflectance as a function of backscattering (\( b_b \)) and absorption (\( a \)) coefficients can be found in the literature. For example, Morel and Gentili\textsuperscript{15} demonstrated that

\[
R_L = \frac{f}{Q} \frac{b_b}{a},
\]

(1)

where the value of \( f/Q \) was found to be variable and dependent on the position of the sensor relative to the direction of incident radiation. Similar models have been developed where the ratio of \( b_b/a \) has been replaced by \( b_b/(a + b_b) \), and the following equation is commonly used for remote sensing interpretation applications:

\[
R_L = l_1 \frac{b_b}{a + b_b} + l_2 \left( \frac{b_b}{a + b_b} \right)^2,
\]

(2)

where \( l_1 \) and \( l_2 \) perform a similar function to \( f/Q \) in Eq. (1).\textsuperscript{16} Various semianalytical models based on Eqs. (1) or (2) have been developed to relate remote sensing reflectance signals to the composition of seawater and other suspended materials for both case 1 and 2 waters.\textsuperscript{17,18}

Equations (1) and (2) both have an implicit dependence on sun-sensor geometry through variability in either \( f/Q \) or \( l_1 \) and \( l_2 \), sometimes referred to as a bidirectional effect.\textsuperscript{19} These parameters also have an implicit dependence on the inherent optical properties (IOPs), including the shape of the VSF.\textsuperscript{20–22}

Mobley et al.\textsuperscript{14} analyzed the effects of scattering phase functions on simulated radiances, irradiances, and reflectances. Their results indicated a sensitivity to the shape of the scattering phase function at intermediate and large scattering angles. However, they suggested that the exact shape of the scattering phase function in the backscatter directions was not critical if errors of the order of 10\% were acceptable. Consequently, it was suggested that the correct backscattering fraction and overall shape of the scattering phase function were crucial for achieving model–data closure. For many applications 10\% may indeed be an acceptable level of error, though current ocean color sensor programs generally set radiometric accuracy of &lt;5\% as their calibration–validation target.\textsuperscript{23–25}

Furthermore Mobley et al.\textsuperscript{14} do not appear to have examined in great detail the impact of structure in the phase function on angularly resolved radiance reflectance (their study was limited to only two solar zenith angles). This study builds upon that of Mobley et al. by (a) investigating in more detail angular structures in modeled light fields, (b) extending the range of IOPs used in simulations to include highly turbid waters, and (c) using newly measured VSF data with genuine levels of angular structure that current analytic phase functions lack.

In this paper we will look at the sensitivity of remote sensing reflectance signals to angular structure in the VSF over a broad range of scattering angles \( \theta \), commensurate with the range of angles encountered in remote sensing applications. For example, Fig. 1 shows the distribution of scattering angles for 2900 medium resolution imaging spectrometer (MERIS) scenes over the bay of Villefranche-sur-Mer, in the Mediterranean (French Riviera). Santer and Martiny\textsuperscript{26} showed a similarly broad range of scattering angles for 50 Sea-viewing Wide Field-of-view Sensor (SeaWiFS) images taken over a sampling site near Venice, Italy. Given this degree of variability in the orientation of Sun-sensor geometry there is clearly a need for detailed study of angular structure in remote sensing reflectance signals. Here we compare angularly resolved reflectances calculated using VSM measured phase functions with those calculated using FF phase functions for given backscattering fractions. Our analysis will extend to IOPs representative of highly turbid waters, and we shall endeavor to develop an improved understanding of the effects of multiple scattering on the remote sensing of backscattered signals for these waters.

2. Materials and Methods

A. In situ Measurements—Volume-Scattering Meter

In situ measurements of the VSF were carried out during a field experiment conducted in the coastal waters of the Black Sea in summer 2004. Here we give a brief description of the experiment and refer to Chami et al.\textsuperscript{9} for more details. The data were collected from an oceanographic platform located 600 m off the southern coast of the Crimean peninsula. Seawater
samples were collected at depths of 0, 4, 8, 12, 16, and 20 m at 1100 and 1400, local time. The VSF was measured using the VSM\textsuperscript{8} (developed by the Marine Hydrophysical Institute, Ukraine, in cooperation with Atlant Inc. Canada). The VSM allows measurements of the VSF from 0.7° to 178° with an angular resolution of 0.3°. The ranges of variation of the scattering coefficient $b_p$, the backscattering coefficient $b_{bp}$, and the backscattering ratio $b_{bp}/b_p$ (defined as the particulate backscattering coefficient divided by the particle scattering coefficient) were 0.1–0.8 m\textsuperscript{-1}, 0.003–0.009 m\textsuperscript{-1}, and 1.0%–2.7% at 443 nm, respectively. The average values of $b_p$, $b_{bp}$, and $b_{bp}/b_p$ were 0.27 ± 0.07 m\textsuperscript{-1}, 0.004 ± 0.0016 m\textsuperscript{-1}, and 1.6% ± 0.3%, respectively.

B. Radiative Transfer Simulations

HYDROLIGHT radiance transfer simulations were performed using a two-component model consisting of water and particles. Simulations were performed at a single wavelength (nominally 443 nm) with IOPs for water taken from the results of Pope and Fry\textsuperscript{27} and Smith and Baker.\textsuperscript{28} Three sets of particulate VSF data from the VSM were converted to particulate scattering phase functions by normalizing to the particulate scattering coefficient ($b_p$—obtained by integrating the VSF), and discretized by using the procedure provided by Sequoia Scientific, Inc.\textsuperscript{29} Simulations performed with these VSM scattering phase functions were compared with runs using FF scattering phase functions with the same particulate backscattering ratio ($b_{bp}/b_p$). As remote sensing reflectance has been shown to be proportional to the ratio of $b_p/a$, the simulations were performed over a range of values of $b_p/a$ from 0.01 to 0.5. Total absorption was held constant at 0.3 m\textsuperscript{-1}, consistent with observations. For each phase function with a given value of particulate backscattering coefficient ($b_{bp}/b_p$), the required range of $b_p/a$ was obtained by varying the particulate scattering coefficient, $b_p$. Simulations were performed with the Sun at zenith, with a nominal wind speed of 5 ms\textsuperscript{-1} to simulate an element of surface roughening, a clear sky, and surface irradiance calculated using the RADTRAN model\textsuperscript{30} supplied as part of the HYDROLIGHT code. A number of simulations were performed at different solar zenith angles. These confirmed that angular reciprocity of the sensor viewing angle relative to the solar zenith angle was maintained across the range of IOP values selected for this analysis.\textsuperscript{31} Thus we were able to use a single solar zenith angle to simulate a wide variety of sensor–solar angles by examining the angular distribution of radiance reflectance, $R_L(\theta) = L_L(\theta)/E_d$. Note that under this convention nadir-viewed reflectance would be described as $R_L(180°)$ and horizontal reflectance would be $R_L(90°)$.

3. Results

Three particulate VSFs measured with the VSM in the Black Sea were selected as being representative of the range of values of $b_{bp}/b_p$ observed during this particular experiment (Fig. 2). These VSFs showed typical structures for natural waters such as strong forward peaks, minima at intermediary angles, and raised levels of scattering toward reverse angles. It is noteworthy that while calculated VSFs from analytic expressions will vary in a systematic manner, measured VSFs are less predictable. Thus although the solid curve in Fig. 2 represents the VSF with the lowest $b_{bp}/b_p$, the magnitude of the VSF signal between 140° and 180° for this sample was greater than that of the sample with $b_{bp}/b_p = 0.018$. Figure 3 shows particulate scattering phase functions obtained by normalizing the VSFs in Fig. 2 to particulate scattering coefficients, $b_p$, together with FF phase functions with the same particulate backscattering ratio. Note that the FF functions were calculated following Mobley et al.\textsuperscript{14} These plots illustrate two significant differences between measured and FF phase functions. First, the VSM phase functions had higher levels of scattering toward 180° than the corresponding FF functions. The range of angles where this began to be true varied from ∼130° to ∼160°. The second major feature was that FF phase functions appeared to be greater than VSM functions at intermediate angles between approximately 10° and 100°. The absolute magnitudes of these differences and the specific angular ranges involved appeared to depend on the particular VSM phase function examined. It is not clear at this point if any correlation could be drawn between the magnitude and nature of the differences between the VSM and FF phase functions and the magnitude of $b_{bp}/b_p$ (and hence perhaps with material composition). $b_{bp}/b_p$ is really used only in this context as an identifying label for each set of phase functions. In this paper our interest is confined to the differences in the angular structure between
the VSM and the FF phase functions with the same \( b_{wp}/b_p \) and the impact this has on simulated values of remote sensing reflectance.

The differences between the VSM and the FF scattering phase functions expressed as a percentage of the VSM functions are plotted in Fig. 4. There were several consistent features among all three sets of phase functions. Between scattering angles of \( \sim 1^\circ \) and \( 3^\circ \), VSM functions were greater than FF functions by a peak value of \( \sim 40\%–50\% \). The difference between the two sets of phase function then decreased to zero at \( \sim 5^\circ \) and then between this angle and \( \sim 120^\circ \) the FF phase functions were up to 60\% greater than those of the VSM. Finally, from 120\° to 180\° the VSM phase functions were greater than or equal to the FF functions with a well-defined peak

Fig. 3. Solid curves represent the particulate scattering phase functions for three values of \( b_{wp}/b_p \) derived from data recorded with the VSM (see Fig. 2). Dashed curves represent FF phase functions with the same backscattering ratio. There are significant differences between VSM and FF phase functions at intermediate and large scattering angles.

Fig. 4. Percentage differences between VSM measured and FF scattering phase functions for angles between (a) 0.7\° and 100\° (logarithm scale for the \( x \) axis), (b) 90\° to 180\° (linear scale for the \( x \)-axis).
difference from 160°–180° with a maximum value of −40%. It is worth noting that these VSM phase functions were derived from data collected on different days at different depths and at different wavelengths. The scattering phase function structures as revealed by VSM measurements and the consistent differences observed between VSM and FF functions suggest that these were significant features of the particulate scattering phase function whose influence on radiances transfer merits further attention.

In this paper we have the benefit of access to VSFs measured across nearly the entire range of scattering angles. This is exceptional because, in general, scattering measurements are collected only at a small number of angles in the backward direction. For example, the WetLabs ECO–VSM measures scattering at three angles (100°, 125°, and 150°), with each measurement averaged over an angular range of 18°. In Fig. 5 we simulate the ECO–VSM data for the three scattering phase functions we have previously examined by taking appropriately weighted VSM measurements (i.e., each simulated value of ECO–VSM data was VSM data at 100°, 125°, and 150° averaged over 18°) and plot this together with equivalent FF phase functions [that might have been derived from \( b_{p}\)/\( b_{p} \), obtained using an ECO–VSM (\( b_{p} \)) and an ac-9 (\( b_{p} \)) instrument for example]. In two cases the simulated ECO–VSM data closely matched the corresponding FF phase functions, but in the third \( b_{p}/b_{p} = 0.025 \) there were strong discrepancies at 100° and 150°. This suggests that ECO–VSM data may be able to give some indication if a FF phase function is not an adequate approximation. However, in all three cases the range of angles measured by the ECO–VSM did not permit observation of significant peaks in scattering beyond 160°, so the method would not be completely reliable. Clearly this is an issue that might be addressed in future generations of commercial in situ optical instrumentation by including an extended range of scattering angles.

HYDROLIGHT simulations were performed for each of the three VSM phase functions described previously and for the corresponding FF phase functions. Subsurface radiances are calculated by dividing radiances signals at each angle, \( L_{s}(\theta) \), by the downward irradiance, \( E_d \). Each set of simulations was performed over a range of \( b_{p}/a \) values, enabling comparison of the effect of differences in the angular structure in the VSF on reflectances for a wide range of scattering levels. Figure 6 shows the differences in radiances reflectance between simulations carried out using VSM and FF scattering phase functions for values of \( b_{p}/a \) ranging from 0.01 to 0.5 and scattering angles ranging from 92.5° to 180°. The magnitude of the difference \( R_{L,VSM} - R_{L,FF} \) can be as large as 0.008 sr⁻¹ (~15% relative difference) which is significant for remote sensing applications [the noise equivalent reflectance of optical instruments is typically approximately 10⁻⁴ sr⁻¹ (Ref. 32)]. Also the impact of variations in the angular structure of the scattering phase function, \( \beta(\theta) \), generally increased for higher values of \( b_{p}/a \) (and hence higher levels of scattering). For low values of \( b_{p}/a (<0.05) \) \( R_{L,VSM} \) was greater than \( R_{L,FF} \) for angles where \( \beta_{VSM}(\theta) > \beta_{FF}(\theta) \) and vice versa, and the differences between \( R_{L,VSM} \) and \( R_{L,FF} \) tended to increase approximately linearly as would be expected in a low-scattering environment where the single-scattering approximation may be valid. However, as \( b_{p}/a \) increased, \( R_{L} \) at all angles tended toward values such that \( R_{L,FF}(\theta) > R_{L,VSM}(\theta) \), even for scattering angles where \( \beta_{VSM}(\theta) < \beta_{FF}(\theta) \). Moreover, even where \( \beta_{VSM}(\theta) < \beta_{FF}(\theta) \) and one would anticipate negative values of \( R_{L,VSM} - R_{L,FF} \) (e.g., at 100°), this parameter increased in magnitude in a nonlinear manner as scattering increased. It is also worth emphasizing that, somewhat unexpectedly, there appeared to be no tendency toward uniformity \( \sum_{i=1}^{3} R_{L,i} = R_{L,FF} \) at high values of scattering where one might assume that multiple scattering would reduce the sensitivity to angular structure in the VSF.

The greatest sensitivity to angular structure was found for the VSM and the FF phase functions with \( b_{p}/b_{p} = 0.025 \). While these phase functions were previously found to have the largest differences in angular structure, we also had to account for the fact that different values of \( b_{p}/b_{p} \) implied different levels
of scattering for a given value of $b_{r}/a$. In Fig. 7 we plot differences between simulations with VSM and FF phase functions expressed as percentages of the VSM radiance reflectance. As absorption was constant for all simulations, this is effectively a normalization with respect to scattering. One observes percentage differences as large as $\pm 20\%$, which happens when $b_{r}/a = 0.03$. As $b_{r}/a$ increased, the percentage differences tended toward constant negative values ranging from 0 (large scattering angles) to $-15\%$ (small scattering angles). Water dominates the IOPs at very low values of $b_{r}/a$, and we could attribute the very rapid increase in the percentage differences between the VSM and the FF simulations of $R_{s}(\theta)$ in this region to a rapid increase in the relative importance of particles over water. However, further analysis would be required in order to understand the nonlinear behavior as $b_{r}/a$ increases.

4. Discussion

The single-scattering approximation alone does not explain the nonlinear relationship between $b_{r}/a$ and the relative differences between $R_{sVSM}$ and $R_{sFF}$ (Figs. 6 and 7). The general nonlinearity of the plot and the fact that FF reflectance is eventually greater than VSM reflectance even at angles where $\beta_{VSM}(\theta)$ is greater than $\beta_{FF}(\theta)$ both have to be explained. Since all IOPs except scattering (and hence backscattering) were kept constant for each set of simulations, we need to examine the possibility that multiple scattering is a key factor in these processes.

A numerical radiance transfer model Ordres Successifs Ocean Atmosphere33 (OSOA) based on the successive order of scattering method was used to calculate radiances at different scattering angles associated with different orders of scattering. Using this method we were able to calculate the radiance associated with each order of scattering (primary, secondary, and so on). The fraction of radiance due to multiple scattering was computed as $1 - L_{1}/L_{tot}$, where $L_{1}$ corresponds to the radiance resulting from single-scattering events, and $L_{tot}$ corresponds to the total radiance. Figure 8 demonstrates the impact of multiple scattering on radiance signals at 90°, 135°, and 170° for VSM and FF phase functions with $b_{r}/a = 0.018$. For values of $b_{r}/a > 0.05$ more than 50% of total radiance was associated with multiple scattering. When $b_{r}/a > 0.3$ the contribution to radiance signals of photons having undergone multiple-scattering events reached as high as 94%. Even at the lowest value of $b_{r}/a = 0.01$, where water dominated the IOPs, almost 20% of the photons contributing to radiance signals at these angles had undergone multiple scattering. Figure 8 suggests that the contribution of multiple scattering to remote sensing reflectance signals increases exponentially with scattering and that the effects of multiple scattering have to be considered for this range of scattering values, as previously suggested by Mobley et al.14 and Park and Ruddick.18 It also highlights that the single-scattering approximation has to
be treated with caution even for clear waters, and is unlikely to hold for waters where $b_w/a > 0.05$.

To further determine the impact of multiple scattering on the radiance reflectance, we will look in more detail at two test cases with different scattering levels ($b_w/a = 0.01$ and $b_w/a = 0.5$) and the same phase function (VSM, $b_{wp}/b_p = 0.018$). For each case we performed Monte Carlo simulations and followed the scattering histories of photons that contributed to the radiance in a particular scattering angle. Here we analyze results for the radiance at a scattering angle of 170° for convenience, but we highlight that the following conclusions hold true when other scattering angles are considered. Thus for radiance at 170° we were able to determine the total number of scattering events $S_{tot}$ that contributed to the radiance signal and also the number of scattering events at angles between 0° and 180° in 2° steps $S_\theta$ that contributed to the radiance signal. Figure 9 shows the fraction of the total number of scattering events $S_\theta$ contributing to $L_w(170)$ for all scattering angles when $b_w/a = 0.01$. As expected there was a strong peak at a scattering angle of 170° corresponding to the contribution of single-scattering photons. It is interesting to note, however, that even with such low levels of scattering, there was also an identifiable contribution from the photons that had been scattered at angles close to zero degrees. The peak of the scattering at forward angles ($\theta < 20°$) was mirrored by the scattering at large angles ($\sim 140°$ to 170°) just outside the 170° bin. It is likely that the

Fig. 7. Differences between VSM and FF simulated radiance reflectances expressed as a percentage of VSM reflectance. The differences reached as high as ±20% and tended toward negative values between 0% and −15% for high turbidity levels. Differences of this magnitude could be significant for remote sensing applications.

Fig. 8. Contribution of multiple scattering to radiance reflectance as a function of the ratio $b_w/a$. The contribution of multiple scattering to radiance reflectance at 90°, 135°, and 170° increased exponentially as the level of scattering increased. The multiple-scattering contribution was calculated using a radiative transfer model based on the successive order of scattering method to calculate total radiance ($L_w$) and radiance from single scattering ($L_1$) at several different angles for VSM and FF phase functions with $b_{wp}/b_p = 0.018$. 

Fig. 9. Contributions of multiple scattering to radiance reflectance as a function of the ratio $b_w/a$. The contribution of multiple scattering to radiance reflectance at 90°, 135°, and 170° increased exponentially as the level of scattering increased. The multiple-scattering contribution was calculated using a radiative transfer model based on the successive order of scattering method to calculate total radiance ($L_w$) and radiance from single scattering ($L_1$) at several different angles for VSM and FF phase functions with $b_{wp}/b_p = 0.018$. 

Y = 0.94 - 0.78 exp (-16x)

$R^2 = 0.996$
main contribution of multiple scattering to a reflected radiance signal at this level of \( b_b/a \) would involve a scattering event at a forward angle, \( \theta_f \), and a scattering event close to the angle of the radiance signal, e.g. \( 170^\circ - \theta_f \) [see the schematic in Fig. 9(b)].

Figure 10 shows the angular distribution of the scattering contributing to the radiance signal at \( 170^\circ \) for \( b_b/a = 0.5 \) and for the VSM phase function with \( b_{bp}/b_p = 0.018 \). Here we see that forward-scattering events played a more significant role than single scattering at \( 170^\circ \) toward the generation of the radiance signal at this angle. In this case, scattering at \( 170^\circ \) contributed only 0.4% of the total number of scattering events that resulted in the generation of the radiance signal at \( 170^\circ \). Under these conditions there may be many different combinations of scattering events that lead to a contribution to radiance at a particular angle. However, it is interesting to note that forward-scattering events had a high probability that was consistent with the shape of the scattering phase function. Therefore the photons that reach the sensor in the upward hemisphere, are preferentially scattered at small angles in turbid waters [Fig. 10(b)].

Our results are consistent with previous theoretical studies,\(^{34,35}\) which demonstrated that the remote sensing reflectance is rigorously dependent on the forward total scattering coefficient, and that the entire shape of the phase function matters. Note that this is the first time to our knowledge that the effects of multiple scattering on reflectance and the contribution of scattering at small angles to the backscattered signal have been quantified.

The effects of the angular structure of the scattering phase function on the reflectance signal for different turbidity levels can be investigated using plots of the angular distribution of scattering events. Figure 11 shows the angular distributions of scattering events contributing to radiance at \( 170^\circ \) for \( b_b/a = 0.01 \) and 0.5, and for two different scattering phase func-
Event contributions to scattering [Fig. 11(a), bb] between these two scattering phase functions. 100°, closely resembling the differences previously observed between VSM and FF simulations for intermediate angles between 10° and 100° (Figs. 3 and 4). This reveals that forward scattering was the dominant contributor to $L_a(170)$ for high $b_o/a$, and therefore the fact that $R_{L,FF}(170) > R_{L,VSM}(170)$ for high $b_o/a$ (see Fig. 6) could be attributed to the fact that the FF phase function was greater than the VSM phase function between 10° and 100°. As this was also true for the other two sets of phase functions ($b_o/a = 0.010$ and 0.025), we can conclude that radiance reflectance signals in highly turbid waters are strongly dependent on the shape of the forward scattering and less dependent on the backscattering structure of the phase function. Based on Figs. 4 and 6, when the value of $b_o/b_p$ is 0.018, a difference in the phase functions of 25% at a scattering angle of 60° can lead to discrepancies in the absolute reflectance $R_L(170°)$ of $2 \times 10^{-3}$ sr$^{-1}$ in turbid waters, which is about one order of magnitude greater than the minimum difference in remote sensing reflectance that can be resolved by usual optical instruments such as satellite sensors or commercial in situ devices.

We can compare our results with those presented by Mobley et al. for the case where $b_o/b_p = 0.018$. In their analysis of phase function shape effects (Section 3 of their paper) they performed radiance transfer simulations using the average Petzold phase function and the Haltrin two-term Henyey–Greenstein phase function. The latter had significantly higher scattering from $-130°$ to $-180°$, and lower scattering at intermediate angles from $-10°$ to $110°$. This was broadly similar in nature to the conditions of our comparison between the VSM and the FF phase functions, although the magnitude of the differences between the phase functions was different in the two analyses. Mobley et al. examined nadir radiance, $L_a(180°)$, for two solar zenith angles (zenith and 60°) and two single-scattering albedos ($\omega_o = b/c = 0.2$ and 0.9). Their results from immediately beneath the surface can be compared with ours for $L_a(180°)$ and $L_a(140°)$, and for $b_o/a = 0.01$ and 0.17, respectively. For low turbidity waters Mobley et al. found $L_a$-Haltrin $(180°) > L_a$-Petzold $(180°)$, and $L_a$-Haltrin $(140°)$ very slightly greater than $L_a$-Petzold $(140°)$, which is consistent with our results (one could substitute VSM for Haltrin and FF for Petzold above). For $\omega_o = 0.9$ Mobley et al. found $L_a$-Haltrin $(140°) = L_a$-Petzold $(140°)$ and $L_a$-Haltrin $(180°) > L_a$-Petzold $(180°)$. This is consistent with the results in Fig. 6(b) where at $b_o/a = 0.17 R_{L,VSM}(180°) > R_{L,FF}(180°)$, and there was a negative trend in the difference between $R_{L,VSM}(140°)$ and $R_{L,FF}(140°)$. However, our analysis differs from that of Mobley et al. as we were able to show that at even higher levels of scattering ($b_o/a = 0.5, \omega_o = 0.96$) such
as those found within river plumes, the reflectance signals corresponding to different phase functions do not converge. In fact, the magnitude of the differences between $R_{L\cdot VSM}$ and $R_{L\cdot FF}$ signals appeared to increase in an approximately linear manner with increasing scattering. Our analysis has shown that, although a broad range of scattering angles do contribute to radiance signals in highly multiple-scattering environments, this does not mean that differences in scattering phase functions average out. In fact, the results of our Monte Carlo simulations (Fig. 11) indicate that differences in phase functions at intermediate angles ($10^\circ$–$110^\circ$) can have a profound impact on reflectance signals in turbid waters.

5. Conclusions

In this paper we have attempted to quantify the impact of the shape of the VSF on simulated radiance reflectance over a wide range of turbidity levels. It has been shown that phase function shape can lead to differences in the estimation of radiance reflectance as high as 20% (Fig. 7) for a given set of absorption, scattering, and backscattering to scattering ratios even in highly scattering waters. Mobley et al.14 and Park and Ruddick15 suggested that multiple scattering should have an influence on the light field. We have demonstrated that this is indeed the case, and that scattering in forward directions is the dominant contributor to reflectance signals in turbid waters. We have also quantitatively estimated, for what we believe to be the first time, the contributions of scattering from different angular portions of the scattering phase function to the remote sensing reflectance. This work suggests that it may be possible to develop improved models of remote sensing reflectance that account for the effects of multiple scattering by either (a) including an additional term to account for the role of the VSF at small and intermediate angles or (b) by splitting $R_{L\cdot}$ into single and multiple scattering components in a manner similar to that developed by Lee et al.22 for molecular and particulate scatterings.

As yet, it is relatively unusual to be able to routinely measure the volume-scattering function with full angular resolution. Therefore it is impractical at this time to state that such measurements ought to be a prerequisite for a successful remote sensing calibration–validation program. Indeed, one should also bear in mind that there may be significant errors in other IOPs used for remote sensing simulations.36,37 In such a context, the assertion by Mobley et al.14 that a Fournier–Forand phase function with the correct backscattering ratio can provide a satisfactory approximation for the true scattering phase function seems quite reasonable. However, it should be borne in mind that this is an approximation and that there may well be significant associated errors. It may be redundant to recommend that resources be put into developing sensors capable of measuring the VSF across a wide range of angles, but perhaps it would be sensible to develop sensors that provided greater angular scattering information than those currently routinely available. Our results show that measurements at intermediate and very large scattering angles should be implemented in future iterations of commercial scattering sensors. This would at least give users an opportunity to assess the validity of FF phase functions derived from measured backscattering ratios.

Appendix A. Notation

- $a$: Total absorption coefficient ($m^{-1}$).
- $b$: Total backscattering coefficient ($m^{-1}$).
- $b_b$: Particulate backscattering coefficient ($m^{-1}$).
- $b_p$: Particulate backscattering coefficient ($m^{-1}$).
- $b_{bp}$: Total backscattering coefficient ratio (i.e., $b_{bp}/b_p$).
- $\beta(\theta)$: Volume-scattering function at the scattering angle $\theta$ ($m^{-1}sr^{-1}$).
- $\beta(\theta)$: Scattering-phase function at the scattering angle $\theta$ ($sr^{-1}$).
- $c$: Total attenuation coefficient ($m^{-1}$) ($c = a + b$).
- $f$: Factor of proportionality between the reflectance and the ratio $b_{bp}/a$.
- $L_u$: Upwelling radiance ($W m^{-2} sr^{-1}$).
- $L_{tot}$: Total radiance ($W m^{-2} sr^{-1}$).
- $L_1$: Radiance at the first order of scattering ($W m^{-2} sr^{-1}$).
- $Q$: Ratio between the upwelling irradiance $E_u$ and the upwelling radiance $L_u$: $Q = E_u/L_u(\theta)(sr)$.
- $\theta$: Scattering angle (degree).
- $\theta_f$: Scattering angle in the forward direction (degree).
- $R_L(\theta)$: Radiance reflectance (in sr$^{-1}$) $L_u(\theta)/E_d(\theta)$, where $L_u$ is the upwelling radiance and $E_d$ is the downwelling irradiance.
- $S_{tot}$: Number of scattering events that contribute to the radiance reflectance.
- $S_u$: Number of scattering events at angle $\theta$ that contribute to the radiance reflectance.
- $\omega_u$: Single-scattering albedo ($\omega_u = b/c$).
- FF: Fournier–Forand phase function.
- IOP: Inherent optical properties.
- MERIS: Medium resolution imaging spectrometer.
- OSOA: Ordres Successifs Ocean Atmosphere.
- VSF: Volume-Scattering Function.
- VSM: Volume-Scattering Meter.

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